

Isochron Dating

Paul Giem

Abstract

The isochron method of dating is used in multiple radiometric dating systems. An explanation of the method and its rationale are given. Mixing lines, an alternative explanation for apparent isochron lines are explained. Mixing lines do not require significant amounts of time to form. Possible ways of distinguishing mixing lines from isochron lines are explored, including believability, concordance with the geological time scale or other radiometric dates, the presence or absence of mixing hyperbolae, and the believability of daughter and reference isotope homogenization. A model for flattening of “isochron” lines utilizing fractional separation and partial mixing is developed, and its application to the problem of reducing the slope of “isochron” lines without significant time is outlined. It is concluded that there is at present a potentially viable explanation for isochron “ages” that does not require significant amounts of time that may be superior to the standard long-age explanation, and that short-age creationists need not uncritically accept the standard long-age interpretation of radiometric dates.

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This paper attempts to accomplish two objectives: First, to explain what isochron dating is and how it is done, and second, to provide an analysis of how reliable it is. In this kind of evaluation, it is important to avoid both over- and underestimates of its reliability. While I will offer tentative conclusions, substantive challenges to those conclusions are welcomed.

Unfortunately, there is no way to deal with the subject without at least mentioning mathematics. This means that math phobics cannot be completely accommodated; they will at least have to see equations. It also means that those who are ignorant of mathematics will need to educate themselves regarding the equations, or else take them on faith.

Introduction

We will begin with the concept of using radiometric dating to measure time. The underlying theory is that a given substance transforms into another, in a process called radioactive decay, at a rate which is proportional to the amount of the initial substance (sometimes called the parent substance). Writing this mathematically, we have

$$dP/dt = -\lambda P,$$

where λ is known as the decay constant and dP/dt is the instantaneous change in P with respect to time. One can integrate this formula¹ to

$$\ln(P_0) - \ln(P) = \ln(P_0/P) = \lambda(t - t_0),$$

where \ln is the natural logarithm, P_0 is the amount of parent substance at a starting time t_0 , and P is the amount of parent substance at any subsequent time t . By convention, we usually define time since t_0 as 0. This makes the equation

$$\ln(P_0/P) = \lambda t.$$

This equation can be transformed into

$$P_0/P = e^{\lambda t},$$

¹ By rearranging, $-dP/P = \lambda dt$. One then integrates, $-\int_{P_0}^P dP/P = \lambda \int_{t_0}^t dt$.

where e (= equals 2.718281 . . .) is the base for natural logarithms. We will see the last two equations and variations a few times in this discussion.

The last two equations, which are equivalent to each other, allow us to date an object, provided that the decay constant λ has not changed and we can measure P and P_0 . The hypothesis that the decay constant has not changed is a reasonable first assumption.² At present, we will assume that the decay constants have not changed, recognizing that this assumption can be challenged.

Measuring P is usually relatively easy. One simply measures the parent element and finds the percentage of the parent element that is of the desired isotope. In practice it is even easier, as almost all the time the percentage of parent that is of the desired isotope is fixed in nature. Thus, for example, in the case of rubidium-strontium dating, rubidium (Rb) in nature consists of 72.15% ⁸⁵Rb (rubidium-85) and 27.85% ⁸⁷Rb (the radioactive isotope). This means that if one wants to determine the amount of ⁸⁷Rb in a given sample, one simply measures the total amount of Rb and multiplies by 0.2785.

Measuring the original amount of parent in the sample, P_0 , is much more complicated. In fact, it is technically impossible, as it would have to be done at the beginning of the time period in question, and obviously for the time periods we are considering that was never done. We can only infer it from other measurements.

The first approximation to P_0 is made by noting that the parent isotope decays to one (or occasionally more) daughter isotopes. If we define D^* as the daughter produced by radioactivity, then

$$P_0 = P + D^*.$$

The problem with this formula is the difficulty of measuring D^* . The daughter product we measure now, D , may not be equal to D^* . There may have been some D at time zero (D_0), there may have been some D added later (D_A), and there may have been some D lost later (D_L). So the correct formula is

$$P_0 = P + D - D_0 - D_A + D_L.$$

² There is some evidence being published that the decay constant may have changed in the past, at least for uranium and lead. However, it is not clear exactly when that happened. Whether it happened during a Flood can be reasonably questioned.

In order for a change in the decay constant to be helpful to a creationist arguing for a short age for fossiliferous strata, the decay constant would have to change during the Flood. However, this would mean that the radioactive elements inside the bodies of Noah, his family, the animals in the ark, and whatever animals survived outside the ark were spared from the otherwise general increase in radioactive decay, or that their bodies proved resistant to the effects of the increased radioactivity. This is not impossible, but does require extra intervention (or more change in the usual laws of physics).

In fact, there may have been some P added later (P_A) and some P lost later (P_L). However, in this case, unless the P is lost or added according to a formula, one cannot even hope to make the necessary corrections, and samples in which loss or gain of parent is suspected are not dated radiometrically, at least when this fact is recognized.

In some cases the last formula is assumed to be enough. For example, in the case of potassium-argon dating, it is assumed that at a given time all the argon in a sample is driven off. One simply measures the daughter (^{40}Ar) isotope, subtracts out air argon, multiplies by a branching factor, and assumes that one then can calculate D^* and therefore P_0 . (A critique of potassium-argon dating is beyond the scope of this paper.) However, in many other methods of radiometric dating the assumption that the daughter isotope is driven off is clearly invalid. For example, ^{87}Sr (strontium-87), the daughter product of ^{87}Rb , is not volatile, and is chemically incorporated into minerals when a melt cools. So the fact that we measure a given amount of ^{87}Sr does not mean that it is a product of decay that accumulated since the rock hardened. It could just as easily have been left in the melt, possibly from previous decay. So we have to have another way to find D^* .

Isochron Lines

The standard way for rubidium-strontium dating, samarium-neodymium dating, lutetium-hafnium dating, potassium-calcium dating, and uranium-lead dating, to name a few dating methods, is to assume isotopic homogenization, or complete mixing. It works something like this in the case of rubidium-strontium dating: At the time of the melt, all the isotopes are assumed to be homogenized. That is, one assumes that initially the isotopic strontium composition was the same throughout a (presumably melted) rock. For example, the $^{87}\text{Sr}/^{86}\text{Sr}$ isotopic ratio³ might be 7.10, or 71 atoms of ^{87}Sr for every 100 atoms of ^{86}Sr . Then the rock crystallized so that the rubidium was partially separated from the strontium. One assumes that there has been no subsequent migration of either rubidium or strontium.

If there is strontium in some mineral without any rubidium, this makes the calculations easy, because this mineral should still have the original $^{87}\text{Sr}/^{86}\text{Sr}$ ratio. Supposing that this ratio was 0.710. That means that if a given rubidium-containing

³ Strontium has three stable isotopes, ^{84}Sr , ^{86}Sr , and ^{88}Sr , which are present in constant ratios relative to each other, so that $^{84}\text{Sr}/^{86}\text{Sr} = 0.056584$ and $^{86}\text{Sr}/^{88}\text{Sr} = 0.1194$, which gives percentages in usual rock of 82.52% ^{88}Sr , 7.00% ^{87}Sr , 9.86% ^{86}Sr , and 0.56% ^{84}Sr . The percentage of ^{87}Sr varies between 6.9% and 7.4%+, depending apparently on the past and/or present rubidium content of the rock. One could use the $^{87}\text{Sr}/^{88}\text{Sr}$ ratio or the $^{87}\text{Sr}/^{84}\text{Sr}$ ratio for our purposes, but the $^{87}\text{Sr}/^{86}\text{Sr}$ ratio is closer to 1, easier to work with, and the traditional one.

mineral now has a $^{87}\text{Sr}/^{86}\text{Sr}$ ratio of 0.720, then for every 1000 atoms of ^{86}Sr , 10 atoms of ^{87}Sr has been produced by radioactivity. If in this mineral the $^{87}\text{Rb}/^{86}\text{Sr}$ ratio is 0.40, then for every 1000 atoms of ^{86}Sr there would be 400 atoms of ^{87}Rb . Thus the original ^{87}Rb concentration would have been 400 + 10, or 410 / 1000 atoms of ^{86}Sr . The formula for the age of the mineral would be

$$t = [\ln (P_0/P)] / \lambda = [\ln (410/400)] \text{ years}/(1.42 \times 10^{11}) = 1.74 \text{ billion years.}$$

If $^{87}\text{Sr}/^{86}\text{Sr}$ is the ratio in the rubidium-containing rock, and $(^{87}\text{Sr}/^{86}\text{Sr})_0$ is the ratio of the rock with no rubidium and therefore the ratio at the time of homogenization, and $^{87}\text{Rb}/^{86}\text{Sr}$ is the ratio in the rubidium-containing rock, then the general formula for the age is

$$t = [\ln ([^{87}\text{Rb}/^{86}\text{Sr} + ^{87}\text{Sr}/^{86}\text{Sr} - (^{87}\text{Sr}/^{86}\text{Sr})_0] / [^{87}\text{Rb}/^{86}\text{Sr}])] / \lambda.$$

The problem with using this formula is that we rarely have a mineral with essentially no rubidium but enough strontium to determine the initial $^{87}\text{Sr}/^{86}\text{Sr}$ ratio. So what is usually done is to obtain several minerals with different degrees of rubidium enrichment so that they have different $^{87}\text{Rb}/^{86}\text{Sr}$ ratios.⁴ Then the $^{87}\text{Rb}/^{86}\text{Sr}$ ratios are plotted against the $^{87}\text{Sr}/^{86}\text{Sr}$ ratios. If these assumptions are correct, we will find our plot giving a straight line.⁵

$$^{87}\text{Sr}/^{86}\text{Sr} = (^{87}\text{Sr}/^{86}\text{Sr})_0 + (e^{\lambda t} - 1) ^{87}\text{Rb}/^{86}\text{Sr}.$$

⁴ The calculations done above can be reversed. That is to say, if the production rate in the given time is one atom of strontium per 41 atoms of rubidium, then for 615 atoms of ^{87}Rb per 1000 ^{86}Sr atoms originally one should have 15 extra atoms of ^{87}Sr now, for a total of 725, and 600 atoms of ^{87}Rb now. This gives different ratios and a different data point, namely, $^{87}\text{Rb}/^{86}\text{Sr} = 0.60$ and $^{87}\text{Sr}/^{86}\text{Sr} = 0.725$

If one starts with more ^{86}Sr , and therefore more ^{87}Sr , one obtains a different ratio. Say that 4000 atoms of ^{86}Sr and therefore 2840 atoms of ^{87}Sr were in the original sample, with 615 ^{87}Rb atoms. Then during this time period, there would still be 15 ^{87}Rb atoms which decayed to ^{87}Sr , for a total of 2855, and 600 ^{87}Rb atoms left. This leaves a $^{87}\text{Rb}/^{86}\text{Sr}$ ratio of 0.15 and a $^{87}\text{Sr}/^{86}\text{Sr}$ ratio of 0.71375. Both of these points, along with the point in the text, are shown on the graph.

⁵ The derivation of the formula is as follows:

$$^{87}\text{Rb}_0 = ^{87}\text{Rb} e^{\lambda t} \quad (\text{Assuming constant decay and no rubidium gain or loss})$$

$$^{87}\text{Rb}_0 = ^{87}\text{Sr}^* + ^{87}\text{Rb} \quad (\text{Decay products})$$

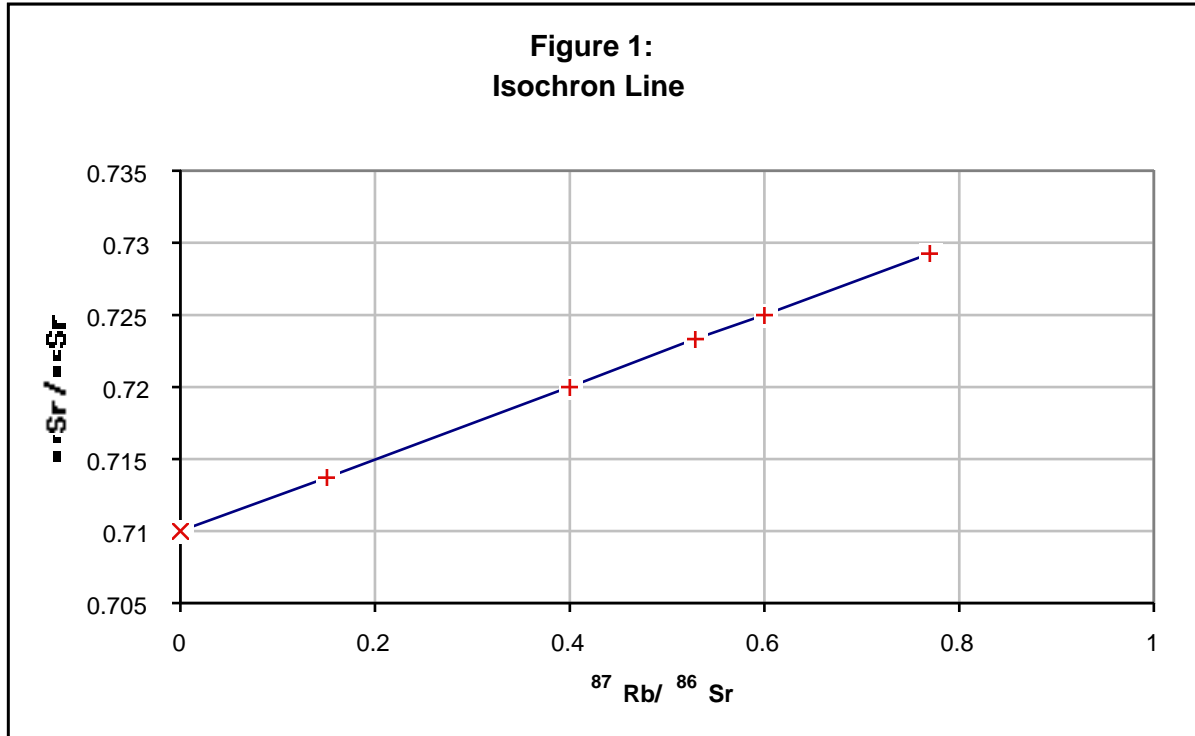
$$^{87}\text{Sr}^* = ^{87}\text{Rb}_0 - ^{87}\text{Rb} = ^{87}\text{Rb} e^{\lambda t} - ^{87}\text{Rb} = ^{87}\text{Rb} (e^{\lambda t} - 1) \quad (\text{Algebra})$$

$$^{87}\text{Sr} = ^{87}\text{Sr}_0 + ^{87}\text{Sr}^* = ^{87}\text{Sr}_0 + ^{87}\text{Rb} (e^{\lambda t} - 1) \quad (\text{Assuming no strontium gain or loss})$$

$$^{87}\text{Sr}/^{86}\text{Sr} = (^{87}\text{Sr}/^{86}\text{Sr})_0 + ^{87}\text{Rb}/^{86}\text{Sr} (e^{\lambda t} - 1) \quad (\text{Assuming isotopic mixing initially})$$

The formula is valid as long as the assumptions are fulfilled.

This is in the form of $y = b + mx$, which makes it a straight line, known as the **isochron line**. The value b gives the intercept, and m gives the slope, which in this case is $(e^{\lambda t} - 1)$. Thus picking some ideal example numbers, we might see a graph like the one below:



Note that where the line crosses the zero line for the $^{87}\text{Rb}/^{86}\text{Sr}$ ratio (marked with an X on the graph) gives the original $^{87}\text{Sr}/^{86}\text{Sr}$ ratio. Any strontium that originally had no rubidium with it would have to have that $^{87}\text{Sr}/^{86}\text{Sr}$ ratio still. Even if there is no such sample, we can predict its composition using our straight line. The apparent age is found by taking the slope,⁶ which in this case is $0.010/0.40$ or 0.025 . So

$$e^{\lambda t} - 1 = 0.025, \quad e^{\lambda t} = 1.025, \quad \text{and} \quad t = \ln(1.025) / k.$$

The general formula is

$$t = \ln(1 + \text{slope}) / k.$$

We only need 2 points to determine the straight line and thus the slope, but if there are more than 2 points, and all our assumptions are correct, the points should all lie on the same straight line.

⁶ Which is the change in the $^{87}\text{Sr}/^{86}\text{Sr}$ ratio divided by the change in the $^{87}\text{Rb}/^{86}\text{Sr}$ ratio.

It is commonly felt that if all the points lie on a straight line, this is a good indication that the above assumptions are correct. For example, see *The Age of the Earth*, a book written by G. Brent Dalrymple,⁷ one of the foremost authorities in the field, with the express purpose of establishing the age of the earth in the face of creationist claims (see the preface). In his discussion of isochron dating, he states (pp. 108-109), “For all practical purposes, the only way to move the isotopic compositions of samples from one isochron to another is by either radioactive decay through time or complete isotopic rehomogenization. Points that fall on an isochron, therefore, can confidently be interpreted as indicating the time of last isotopic homogenization, i. e. formation or reformation of the rock. Thus the isochron method is self-checking, providing not only the prospect of an age but also a statement on its validity.”

Mixing lines

However, this confident statement is an overstatement. There is a process which can routinely give a straight line on an isochron plot, while having essentially nothing to do with time. It is called a mixing line. If two rocks are mixed in varying proportions, and the $^{87}\text{Rb}/^{86}\text{Sr}$ ratio is plotted against the $^{87}\text{Sr}/^{86}\text{Sr}$ ratio, the result is always a straight line. The derivation is as follows.

We will take two rocks, rock 1 and rock 2. Rock 1 contains p_1 parent, d_1 daughter, and r_1 reference isotope (in the case of rubidium-strontium dating, p is ^{87}Rb , d is ^{87}Sr , and r is ^{86}Sr). Rock 2 contains p_2 parent, d_2 daughter, and r_2 reference isotope (all of these are concentrations). Then we will mix them in proportion so that the proportion of our final rock that is rock 1 is a and the proportion of our final rock that is rock 2 is b . We have

$$a + b = 1,$$

that is, all the final rock is either a or b . Now we will assume that the rocks are not identical, and that both rocks have some r , so that p/r and d/r have some meaning at the endpoints. (If $r = 0$ at any point, p/r and d/r have no meaning at that point, and we cannot draw an isochron line either). We now have

$$p = ap_1 + bp_2$$

$$d = ad_1 + bd_2$$

$$r = ar_1 + br_2$$

⁷ Stanford, CA: Stanford University Press, 1991.

(The first equation says that the concentration of parent in the final rock is the concentration from rock 1 times the proportion we used, plus a similar component from rock 2)

$$d/r = (ad_1 + bd_2) / r$$

$$\begin{aligned} (p_1r_2 - p_2r_1) d/r &= (p_1r_2 - p_2r_1) (ad_1 + bd_2) / r \\ &= (p_1r_2ad_1 + p_1r_2bd_2 - p_2r_1ad_1 - p_2r_1bd_2) / r \\ &= (ap_1d_1r_2 + bp_1d_2r_2 - ap_2d_1r_1 - bp_2d_2r_1) / r \\ &= (ap_1d_1r_2 + bp_1d_2r_2 - ap_2d_1r_1 - bp_2d_2r_1 + ap_1d_2r_1 - ap_1d_2r_1 + bp_2d_1r_2 - bp_2d_1r_2) / r \\ &= (ap_1d_1r_2 - ap_1d_2r_1 + bp_2d_1r_2 - bp_2d_2r_1 + ap_1d_2r_1 - ap_2d_1r_1 + bp_1d_2r_2 - bp_2d_1r_2) / r \\ &= (ap_1(d_1r_2 - d_2r_1) + bp_2(d_1r_2 - d_2r_1) + ar_1(p_1d_2 - p_2d_1) + br_2(p_1d_2 - p_2d_1)) / r \\ &= ((ap_1 + bp_2)(d_1r_2 - d_2r_1) + (ar_1 + br_2)(p_1d_2 - p_2d_1)) / r \\ &= (p(d_1r_2 - d_2r_1) + r(p_1d_2 - p_2d_1)) / r \end{aligned}$$

$$d/r (p_1r_2 - p_2r_1) = (p/r) (d_1r_2 - d_2r_1) + (p_1d_2 - p_2d_1)$$

which is in the form $By = -Ax + C$ or $Ax + By = C$, which is a straight line, **always**. If p_1/r_1 does not equal p_2/r_2 (that is, the two rocks do not have the same proportion of parent to reference isotope), we may divide by $p_1r_2 - p_2r_1$, and

$$d/r = (p/r) (d_1r_2 - d_2r_1) / (p_1r_2 - p_2r_1) + (p_1d_2 - p_2d_1) / (p_1r_2 - p_2r_1),$$

which is of the form $d/r = S p/r + Y$, where S is the slope and Y is the y-intercept. A mixing line not only can, but always does precisely mimic an isochron plot. This fact is well known in the field.

It is important to realize that a mixing line does not require thorough mixing of the two components, as long as we are doing whole rock dating. All that is required is that each component itself is homogeneous. One can even create a mixing line by, for example, taking a piece off of each component and simply putting those two pieces in a specimen container, measuring the relevant isotopes, and repeating the process. As long

as the pieces do not have precisely the same ratio of rock 1 to rock 2, we will get a mixing line. Any scatter in the line is evidence for gross post-mixing fractionation, contamination, and/or leaching, not incomplete mixing. One cannot tell a mixing line from a true isochron line on the basis of its straightness.

Distinguishing mixing lines from isochrons

The question can be raised as to how sure we can be that a given line is an isochron plot rather than a mixing line. There are two ways. First, the date has to be believable on other grounds. Second, it has to be believable that the strontium isotopes in a given rock have indeed homogenized at the time.

Of course, the first criterion immediately raises questions. Believable by whom? Believable for what reasons? Some may be tempted to question whether such criteria are ever used. However, it appears that they are. Gunter Faure, in his classic book *Principles of Isotope Geology*,⁸ lists several rubidium-strontium dates that do not match the standard geologic time scale and are confidently written off as mixing lines without further evidence, at least as far as I can tell.⁹ The same holds true for “futurechrons”, apparent isochron lines that date in the future.¹⁰

Of course, this raises at least the possibility that if a creationist did not believe the standard geologic time scale, he/she is not obligated to believe any of the rubidium-strontium dates in the literature, or any of the other dates that are “confirmed” by the isochron method. This leaves us in an uncomfortable position. Science does not like to leave matters where either of two widely disparate theories are compatible with exactly

⁸ 2nd ed. New York: John Wiley and Sons., 1986

⁹ Pp. 145-7. His examples follow: Pleistocene to Recent (<1.6 million years old) lava with a Rb/Sr age of 773 million years (Bell K, Powell JL: “Strontium isotopic studies of alkalic rocks: The potassium-rich lavas of the Birunga and Toro-Ankole Regions, east and central Africa.” *J Petrol* 1969;10:536-72); upper Miocene to Pliocene (5-9 million years old by K/Ar dating) lava with a Rb/Sr age of 31-39 million years (Dickinson DR, Dodson Mn, Gass IG, Rex DC: “Correlation of initial ⁸⁷Sr/⁸⁶Sr with Rb/Sr in some late Tertiary volcanic rocks of south Arabia.” *Earth Planet Sci Lett* 1969;6:84-90); Pliocene to Holocene (<5.3 million years old) lava giving Rb/Sr ages of 570 and 870 million years (the 570 million year “isochron” is apparently from <3000 year old lava). Leeman WP, Manton WI: “Strontium isotopic composition of basaltic lavas from the Snake River Plain, southern Idaho.” *Earth Planet Sci Lett* 1971;11:420-34); and Miocene to Holocene (<24 million years old) volcanic rock with a Rb/Sr age of 1.2 billion years (Duncan RA, Compston W: “Sr-isotopic evidence for an old mantle source region for French Polynesian volcanism.” *Geology* 1976;4:728-32). An additional report has been made of Pliocene to Holocene (<5.3 million years old) lava with a Rb/Sr age of 1.5 billion years (Leeman WP: “Late Cenozoic alkalirich basalt from the western Grand Canyon area, Utah and Arizona: Isotopic composition of strontium.” *Bull Geol Soc Am* 1974;85: 1691-6).

¹⁰ For an example, see Dasch EJ, Green DH: “Strontium isotope geochemistry of lherzolite inclusions and host basaltic rocks, Victoria, Australia.” *Am J Sci* 1975;275:461-9.

the same phenomena. Hopefully somewhere they will make differing predictions, and can thus be distinguished by testing.

Concordance with the geological time scale or other radiometric dates

One might suppose that having most of the dates match the standard geologic time scale would be a prediction of evolutionary theory, whereas short-age creationists would expect the ages not to match. However, it is known that some but not all of the dates match. What percentage is enough to validate the standard geologic time scale? 95%? 60%? 30%? 5%? And how do you determine the percentage? By the statistical limits of error? By being within 20% of the predicted value? And do all published data count, or the raw data from a sample of laboratories? I personally think the project would be a fascinating one. But at the end of the day (or rather, project) I doubt that either side would concede based on this evidence.

One might also suppose that if several different dating methods all got the same age, this adds credibility to the date obtained. There are three problems with this approach. First, one must then subtract credibility if the methods disagree. This is not always done. Second, it is predictable that if two magmas that have been either naturally or artificially aged to the same degree are mixed, all their “isochron lines” will match without the match having anything to do with time since deposition, even if the magmas in question have been melted.¹¹ Finally, there are multiple references in the literature to matching radiometric dates that disagree with the standard geologic assigned age.¹²

Mixing hyperbolae

One can approach the problem another way. Mixing lines are supposed to give mixing hyperbolae. We will try to address the question whether mixing hyperbolae can reliably distinguish mixing lines from isochrons. For completeness, the technical details are given, but since they are, or at least should be, non-controversial, they will be given in smaller print (some readers may wish to skip over them to the conclusion).

¹¹ To illustrate, supposing that one originally had two magmas, both with the same $^{87}\text{Sr}/^{86}\text{Sr}$ ratio and the same $^{143}\text{Nd}/^{144}\text{Nd}$ ratios but differing contents of ^{87}Rb and ^{147}Sm . If one “aged” the two magmas, part of the ^{87}Rb would change to ^{87}Sr , and part of the ^{147}Sm would change to ^{143}Nd . If one mixed the two magmas, one would then get straight lines which would appear to be isochron lines precisely matching in “age”, while the mixing could have been done fifteen minutes before the measurement.

Basically, the two magmas would behave as two points of a true isochron line with respect to each isotope system. Since they are aged in the same way, they will give the same dates. The mathematics of mixing will guarantee that the various mixtures of the two magmas will fall along the same line as the magmas themselves, thus giving the same dates as if one simply measured the two magma sources and used the line determined by their two points for the date.

For concordant flattening of “isochrons” which are really mixing lines, see below.

¹² For example, see Odin GS (ed): *Numerical Dating in Stratigraphy*. Chinchester, UK: John Wiley and Sons, 1982, especially chapters 12 and 24.

What is a mixing hyperbola? We need to note first that most plots on a two-component mixture (TCM) are straight lines. If one plots the amounts of any two substances x and y in a given TCM, one gets a straight line. That is because for any given component x we have the concentration of x expressed by

$$x = ax_1 + (1-a)x_2,$$

where a is the fraction of a given rock that is from rock 1, x_1 is the concentration of x in rock 1, and x_2 is the concentration of x in rock 2. This equation can be further manipulated:

$$x = x_2 + a(x_1 - x_2),$$

or, if one assumes that x_1 does not equal x_2 (which we will routinely assume—we are interested only in variable components),

$$a = (x - x_2)/(x_1 - x_2).$$

For a second component y we have

$$a = (y - y_2)/(y_1 - y_2).$$

We thus have

$$\begin{aligned} (x - x_2)/(x_1 - x_2) &= (y - y_2)/(y_1 - y_2), \\ y - y_2 &= [(y_1 - y_2)/(x_1 - x_2)](x - x_2), \text{ and} \\ y &= [(y_1 - y_2)/(x_1 - x_2)]x - x_2 [(y_1 - y_2)/(x_1 - x_2)] + y_2, \end{aligned}$$

which is of the form $y = mx + b$, which is a straight line.

If one plots x/z versus y/z for a TCM, one also obtains a straight line. The derivation of this was given above, if one substitutes p for x , d for y , and r for z . And if one plots x/w versus y/z , where there is some constant k such that $w = kz$, one still gets a straight line. One can simply define a new variable $v = kx$, and use the same derivation, as v/w now becomes $kx/kz = x/z$, which was shown above to give a straight line. If v/w versus y/z gives a straight line, then kx/w versus y/z gives the same straight line, and x/w versus y/z gives a straight line also, only with a changed slope. (This means that a TCM will always give a straight line on a discordia plot.)

But if one plots x versus y/z , one now gets a hyperbolic plot, assuming that z_1 does not equal z_2 . What do we mean by a hyperbolic plot? It is a plot where a variable, say x , is inversely proportional to another variable, say y , so that in the simplest case

$$y = c/x, \text{ (} c \text{ being a constant),}$$

or in a more complicated case

$$\begin{aligned} y - c_1 &= c_2/(x - c_3), \text{ or} \\ y &= c_1 + c_2/(x - c_3). \end{aligned}$$

In this case c_1 determines the distance from the origin to the y - (conventionally horizontal) asymptote, c_3 determines the distance from the origin to the x - (conventionally vertical) asymptote, and c_2 determines the distance from the crossing of the asymptotes to the curve itself.

The reason we have a hyperbolic plot for x versus y/z is because on the x -axis we have

$$z = m_1x + b_1,$$

which means that z is linearly dependent on x . At the same time

$$y = m_2z + b_2.$$

This means that

$$y/z = (m_2z + b_2)/z = m_2 + b_2/z,$$

which plots against z as a hyperbola.

But it also plots against x as a hyperbola, for

$$y/z = m_2 + b_2/(m_1x + b_1) = m_2 + (b_2/m_1)/(x + b_1/m_1),$$

which is of the form $y/z = c_1 + c_2/(x - c_3)$.

For x/w versus y/z (assuming z is not proportional to w), the math is more complicated, but the result is the same. Solving the relationships between y/z and w , we have

$$y/z = m_2 + b_2/(m_1 w + b_1) = m_2 + (b_2/m_1)/(w + b_1/m_1) \text{ and}$$

$$x = m_3 w + b_3, \text{ so that}$$

$$x/w = m_3 + b_3/w.$$

This means that

$$w = b_3/(x/w - m_3).$$

Therefore

$$y/z = m_2 + b_2/(m_1 w + b_1) = m_2 + (b_2/m_1)/([b_3/(x/w - m_3)] + b_1/m_1)$$

$$= m_2 + (x/w - m_3)(b_2/m_1)/(b_3 + (x/w - m_3)b_1/m_1)$$

$$= m_2 + (x/w - m_3)(b_2/b_1)/(b_3 m_1/b_1 + x/w - m_3)$$

$$= m_2 + [(x/w)(b_2/b_1) - m_3(b_2/b_1)]/(x/w + b_3 m_1/b_1 - m_3)$$

$$= m_2 + [(x/w)(b_2/b_1) + (b_3 m_1/b_1 - m_3)(b_2/b_1) - (b_3 m_1/b_1 - m_3)(b_2/b_1) - m_3(b_2/b_1)]/(x/w + b_3 m_1/b_1 - m_3)$$

$$= m_2 + b_2/b_1 - (b_3 m_1/b_1)(b_2/b_1)/(x/w + b_3 m_1/b_1 - m_3),$$

which is of the form $y/z = c_1 + c_2/(x - c_3)$.

The point of all this is that both the straight lines and the mixing hyperbolas are direct consequences of the linear relationships of various components to each other. Although a mixing line is the easiest way to produce this relationship, any process that will produce the same relationship will produce mixing plots.

Suppose one had a true isochron plot. If it is a 2-component plot, standard theory holds that either it is a two-component mixture (TCM) where the strontium isotopes have equilibrated, or it is a completely homogenized (presumably melted) rock which has separated into two phases, presumably by crystallization. If it is a TCM, then the distributions of all components including rubidium and strontium (or other parent isotopes and daughter product) are linear, the distributions of parent and daughter isotopes should stay linear while decay is taking place, and all plots should be indistinguishable from mixing lines and/or hyperbolae. If it crystallized from a melt into two phases, we have the mathematical equivalent of a TCM, and the plots should again be indistinguishable from mixing lines and/or hyperbolae. So isochrons in a two-component system are not distinguishable from a two-component mixing line.

One is tempted to suggest that perhaps the differentiation between a mixing line and an isochron line is easier if there are 3 (or 4 or more) components to the rock suite. A mixing line with more than two components should not give a straight line on an isochron plot, and a rock suite isochron with more than 2 mineral types should not give mixing hyperbolae (because most other components should not have straight lines colinear with the isochron line components). So if we find a 3 (or more) mineral suite of rocks with an "isochron" line, it should be unusual for it to be a mixing line, and we should be able to tell the difference by seeing whether mixing hyperbolae are present. Mixing lines should have them, and isochrons should not.

However, there are several problems with this way of differentiating mixing lines from isochron lines. First, minor components may add too little mass and/or amounts of the minerals being measured to change either the "isochron" line or the mixing

hyperbola. Alternatively, the other components might be too evenly distributed in the rock to change the essential mathematics. So a mixing line might still be a good straight imitation of an isochron plot, and a true isochron might still show a good “mixing hyperbola”.

Second, if the third (and fourth or more if present) component of a mixing line has, for example, a $^{87}\text{Rb}/^{86}\text{Sr}$ ratio that plots against its $^{87}\text{Sr}/^{86}\text{Sr}$ ratio on a straight line with the respective ratios of the first two components, one will still get a straight “isochron” line which is really a mixing line. In fact, the other components do not even have to be exactly on the line. They only have to be close enough so that the confidence limits of their measurements overlap the line. In this case we can have a mixing line without necessarily having good mixing hyperbolae, mimicking an isochron plot.

In fact, according to Faure (p. 151) “However, suites of samples formed by mixing of two components may not fit a single mixing equation because the end members may have had variable isotopic and chemical compositions. Moreover, both chemical and isotopic compositions of rocks may be changed by processes occurring *subsequent* to mixing, such as fractional crystallization, contamination by third components, and alteration by hydrothermal fluids or chemical weathering. Therefore a certain amount of scatter of data points above and below the mixing equation is commonly observed for suites of geologic samples that are in fact binary mixtures.” (italics his) Thus Faure is not confident that a two-component mixture will always give good mixing hyperbolae.

Finally, the third (and fourth or more if present) component of a true cogenetic suite of rocks (giving a true isochron) might happen to have, for example, their Rb/Sr ratios roughly colinear with their absolute Sr concentrations. In that case they will also give “mixing hyperbolae” even though they are not mixing lines.

It will be theoretically interesting to test mixing lines for mixing hyperbolae, and also lines that are considered true isochrons by long-age geochronologists, and see what proportion of each has mixing hyperbolae. To my knowledge this has not been done. I am working on getting that done on at least a few samples. However, the results are not likely to be regarded as conclusive by either side.

Believability of isotopic homogenization

The final possible difference between the short-age and long-age interpretations of isochron dating focuses on a crucial assumption. Is it believable that, for example, the strontium isotopes have homogenized?

Intuitively, it would seem that strontium isotopes should be easier to homogenize within the minerals of a given rock specimen than across rock suites. This is especially true if the rock suites extend across kilometers. So when rubidium-strontium dating is done, one might expect that minerals within a given rock would be used preferentially.

However, this is apparently not usually the case. One of the statements that stunned me when I first read about Rb-Sr dating was found in Faure as his first paragraph on experimental results (pp. 120-121):

Igneous rocks of granitic composition may contain both mica minerals and K-feldspar, all of which can be dated by the Rb-Sr method. Ideally, all minerals of an igneous rock should indicate the same date which can then be regarded as the age of the rock. When mineral dates obtained from one rock specimen or from a suite of cogenetic igneous rocks are in agreement, they are said to be “concordant.” Unfortunately, “discordance” of mineral dates is more common than “concordance.” The reason is that the constituent minerals of a rock may gain or lose radiogenic ^{87}Sr as a result of reheating during regional or contact metamorphism after crystallization from a magma. In such cases, the mineral dates generally are not reliable indicators of the age of the rock. We must therefore turn to the rocks themselves if we want to determine their ages.

It is much easier for me to visualize equilibration of strontium isotopes in the minerals in a rock than in whole rock samples in a batholith, for example. Unfortunately, we have almost no direct experimental evidence here. The indirect evidence I can find (admittedly not exhaustive, and for that reason very possibly not representative, but reasonably representative of the evidence I have seen), is in four forms:

1. Sedimentary rocks, deposited under water, do not homogenize their strontium if the grain size, at least of illite clay, is 2 microns or larger (and maybe not then—the limit is based on the assumption that long-age dates are correct).¹³

2. Whole rock dating is sometimes assumed to be reasonably accurate when the whole rocks are separated by miles. In fact, in one case, because the Rb-Sr dates are younger than the other dates, it is assumed that without further physical mixing, the strontium isotopes somehow equilibrated across the same distances after the rock was emplaced. And this without affecting the argon content of the rocks, so that the K-Ar age was still correct!¹⁴ I find item 2 difficult to believe, but frankly incredible when comparing it with item 1. I don't see how one can have it both ways.

3. There are several examples of lava which flowed in recent to Miocene times whose Rb-Sr dates are in the 500-1,500 Ma range.¹⁵

4. The experimental data are against easy migration of strontium atoms, at least according to Hanson and Gast.¹⁶ On p. 1120 they stated, “It is significant that no one has

¹³ Faure, p. 130, cites some examples.

¹⁴ Lanphere MA, Wasserburg GJF, Albee AL, Tilton GR: “Redistribution of strontium and rubidium isotopes during metamorphism, World Beater Complex, Panamint Range, California.” In: Craig H, Miller SL, Wasserburg GJ (eds): *Isotopic and Cosmic Chemistry*. Amsterdam: North-Holland Publishing Company, 1964, pp. 269-320.

¹⁵ See note 8 for some examples

so far been able to thermally induce radiogenic strontium-87 to leave its host mineral in quantities commensurate to the loss of argon under geologically reasonable conditions even though it is not uncommon to find biotites in nature which have lost both radiogenic argon-40 and strontium-87 due to a thermal event.” I have not seen any data that would challenge their conclusion.

For what it is worth, where I have seen the mineral dates reported, they have generally been about an order of magnitude younger than the corresponding whole rock dates.

It would appear from the foregoing that the theory of strontium isotopic equilibration does not have the support of the available experimental evidence. This should be tested, as it may make it possible to distinguish experimentally between the predictions of short-age and long-age interpretations of life on earth, at least if short-age theories assume no significant change in radioactive time constants during the Flood. Further evaluation may also clarify the validity of other methods of isochron dating.

Should all mixing lines from similarly “aged” rocks give the same date?

If all the magma on the earth is “aged” (either naturally or artificially) to the same extent, would one not get the same age for all isochrons, unless some are true isochrons? In other words, if all magmas were initially to lie on the same isochron line, would not all mixtures of these magmas give either points or “isochron” lines with identical “ages” when they are mixed?

If the geology of earth were simple, this would be the case. However, it is not simple, and several kinds of processes can flatten the original isochron without requiring millions of years. For example, let us take two magmas, one with a $^{87}\text{Rb}/^{86}\text{Sr}$ ratio of 0.64 and a $^{87}\text{Sr}/^{86}\text{Sr}$ ratio of 0.70, and one with a $^{87}\text{Rb}/^{86}\text{Sr}$ ratio of 0.16 and a $^{87}\text{Sr}/^{86}\text{Sr}$ ratio of 0.70. The two magmas have the same $^{87}\text{Sr}/^{86}\text{Sr}$ ratio, as they are primordial. We then age the two magmas 4.54 billion years (naturally or artificially—it does not matter which). We now have a $^{87}\text{Rb}/^{86}\text{Sr}$ ratio of 0.60 and a $^{87}\text{Sr}/^{86}\text{Sr}$ ratio of 0.74 for the first magma and a $^{87}\text{Rb}/^{86}\text{Sr}$ ratio of 0.15 and a $^{87}\text{Sr}/^{86}\text{Sr}$ ratio of 0.71 for the second magma. We now send both magmas up through country rock with a composition identical to the second magma. The composition of the second magma does not change. However, if the first magma dissolves enough country rock so that the ^{86}Sr content of the country rock is equal to the ^{86}Sr content of the first magma, the new magma will have a $^{87}\text{Rb}/^{86}\text{Sr}$ ratio of 0.375 and a $^{87}\text{Sr}/^{86}\text{Sr}$ of 0.725. This would still be on the original isochron line. But if we allow the magma to fractionally crystallize so that the $^{87}\text{Rb}/^{86}\text{Sr}$ ratio of the more fluid part rises

¹⁶ Hanson GN, Gast PW: “Kinetic studies in contact metamorphic zones.” *Geochim et Cosmochim Acta* 1967;31:1119-53.

back to 0.60,¹² the $^{87}\text{Sr}/^{86}\text{Sr}$ ratio should not change. This magma may now mix with either the second magma or the country rock, and the mixing line that is created will have a slope of 1/30, with an “age” of 2.31 billion years. We have just cut the “age” almost in half with a perfectly straight “isochron” line, all with essentially no time required. This example does not appear unreasonable.

So the answer to the last question is, one can fairly easily flatten “isochrons” that are really mixing lines, and give straight lines, as long as one allows for mixing and fractional crystallization. It does not require large amounts of time.

What about multiple “isochrons”? Can they be flattened in this way?

If there is a characteristic fractionation coefficient for each of several isochron dating systems, and some process of magma mixing and re-differentiation occurs, the same percentage flattening can happen with each system. One therefore can obtain matching reduced “isochron” dates, which actually are produced by mixing lines. Whether this process is believable depends on the precision of the measurements and the precise parameters of mixing. As far as I know, one cannot find this derivation in textbooks, and so those with the proper competence are urged to follow the math closely looking for errors.

Suppose we take a rock and differentiate it into two fractions, fraction 1 containing p_1 parent, d_1 daughter, and r_1 reference isotopes, and fraction 2 containing p_2 parent, d_2 daughter, and r_2 reference isotopes. We note that

$$d_1/r_1 = d_2/r_2,$$

but assume that

$$p_1/r_1 \neq p_2/r_2$$

(so that the rock can be dated). This means that

$$d_1 r_2 = d_2 r_1, \text{ and } d_1 r_2 - d_2 r_1 = 0$$

and that $p_1 r_2 - p_2 r_1 \neq 0$.

¹² We have assumed (a reasonable first approximation) that the ratio of rubidium in the rubidium-rich fraction to rubidium in the rubidium-poor fraction has remained constant. We have also assumed that the ratio of strontium in the rubidium-rich fraction to strontium in the rubidium-poor fraction has not changed. Finally, we have assumed that the volume of rubidium-rich rock and that of rubidium-poor rock have not changed. Slightly different numbers will be obtained if we assume that the numbers can change, but since rubidium is a very tiny fraction of the rock, we should not expect the changes to be great.

In fact, the situation is more complicated. An adjustment should be made for the fact that there is now more calcium and less potassium than previously. The potassium will change by roughly one part in 1,000, and the calcium (on the average) by somewhat less. Other elements will also change slightly. At some point making the corrections becomes not worthwhile.

We now age it (either naturally or artificially) so that a fraction f of the parent in each rock turns into daughter. We now have, for rock 1, $p_1 - fp_1$ parent and $d_1 + fp_1$ daughter, and for rock 2 $p_2 - fp_2$ parent and $d_2 + fp_2$ daughter. The slope of the line is

$$\frac{(d_1 + fp_1)/r_1 - (d_2 + fp_2)/r_2}{(p_1 - fp_1)/r_1 - (p_2 - fp_2)/r_2}$$

or

$$\frac{d_1 r_2 + fp_1 r_2 - d_2 r_1 - fp_2 r_1}{p_1 r_2 - fp_1 r_1 - p_2 r_1 + fp_2 r_2}$$

or

$$\frac{d_1 r_2 - d_2 r_1 + fp_1 r_2 - fp_2 r_1}{p_1 r_2 - p_2 r_1 - fp_1 r_1 + fp_2 r_2}$$

or

$$\frac{d_1 r_2 - d_2 r_1 + f(p_1 r_2 - p_2 r_1)}{1(p_1 r_2 - p_2 r_1) - f(p_1 r_1 + p_2 r_2)}$$

or (since $d_1 r_2 - d_2 r_1 = 0$)

$$f/(1-f).$$

This makes sense, for since $t = \ln(p_0/p) / k$, and

$$p_0/p = p_0/(p_0 - fp_0) = 1/(1-f) = (1-f+f)/(1-f) = 1-f/(1-f) = 1 - \text{slope},$$

then $t = \ln(1 - \text{slope}) / k$, which is the traditional formula for the age of an isochron.

We now mix a unit amount of fraction 1 with x amount of fraction 2, and a unit amount of fraction 2 with y amount of fraction 1. The new concentrations will be, for parent,

$p_{1 \text{ mixed}} = [p_1 - fp_1 + x(p_2 - fp_2)] / (1 + x)$ and $p_{2 \text{ mixed}} = [p_2 - fp_2 + y(p_1 - fp_1)] / (1 + y)$
for daughter,

$d_{1 \text{ mixed}} = [d_1 + fp_1 - x(d_2 + fp_2)] / (1+x)$ and $p_{2 \text{ mixed}} = [d_2 + fp_2 + y(d_1 + fp_1)] / (1 + y)$
and for reference isotope,

$$r_{1 \text{ mixed}} = (r_1 + xr_2) / (1 + x) \text{ and } r_{2 \text{ mixed}} = (r_2 + yr_1) / (1 + y)$$

The slope for this line is

$$\frac{\frac{[d_1 + fp_1 + x(d_2 + fp_2)] / (1 + x)}{(r_1 + xr_2) / (1 + x)} - \frac{[d_2 + fp_2 + y(d_1 + fp_1)] / (1 + y)}{(r_2 + yr_1) / (1 + y)}}{\frac{[p_1 - fp_1 + x(p_2 - fp_2)] / (1 + x)}{(r_1 + xr_2) / (1 + x)} - \frac{[p_2 - fp_2 + y(p_1 - fp_1)] / (1 + y)}{(r_2 + yr_1) / (1 + y)}}$$

or

$$\frac{[d_1 + fp_1 + x(d_2 + fp_2)](r_2 + yr_1) - [d_2 + fp_2 + y(d_1 + fp_1)](r_1 + xr_2)}{[p_1 - fp_1 + x(p_2 - fp_2)](r_2 + yr_1) - [p_2 - fp_2 + y(p_1 - fp_1)](r_1 + xr_2)}$$

or

$$\begin{aligned} & d_1r_2 + fp_1r_2 + xd_2r_2 + xfp_2r_2 + yd_1r_1 + yfp_1r_1 + xyd_2r_1 + xyfp_2r_1 \\ & \underline{-d_2r_1 - fp_2r_1 - yd_1r_1 - yfp_1r_1 - xd_2r_2 - xfp_2r_2 - xyd_1r_2 - xyfp_1r_2} \\ & p_1r_2 - fp_1r_2 + xp_2r_2 - xfp_2r_2 + yp_1r_1 - yfp_1r_1 + xyp_2r_1 - xyfp_2r_1 \\ & -p_2r_1 + fp_2r_1 - yp_1r_1 + yfp_1r_1 - xp_2r_2 + xfp_2r_2 - xyp_1r_2 + xyfp_1r_2 \end{aligned}$$

or

$$\begin{aligned} & d_1r_2 + fp_1r_2 + (yd_1r_1 - yd_1r_1) + (yfp_1r_1 - yfp_1r_1) + xyd_2r_1 + xyfp_2r_1 \\ & \underline{-d_2r_1 - fp_2r_1 + (xd_2r_2 - xd_2r_2) + (xfp_2r_2 - xfp_2r_2) - xyd_1r_2 - xyfp_1r_2} \\ & p_1r_2 - fp_1r_2 + (yp_1r_1 - yp_1r_1) + (yfp_1r_1 - yfp_1r_1) + xyp_2r_1 - xyfp_2r_1 \\ & -p_2r_1 + fp_2r_1 + (xp_2r_2 - xp_2r_2) + (xfp_2r_2 - xfp_2r_2) - xyp_1r_2 + xyfp_1r_2 \end{aligned}$$

or

$$\begin{aligned} & \underline{d_1r_2 - d_2r_1 + xyd_2r_1 - xyd_1r_2 + fp_1r_2 - fp_2r_1 + xyfp_2r_1 - xyfp_1r_2} \\ & p_1r_2 - p_2r_1 - fp_1r_2 + fp_2r_1 - xyp_1r_2 + xyp_2r_1 + xyfp_1r_2 - xyfp_2r_1 \end{aligned}$$

or

$$\begin{aligned} & \underline{f(p_1r_2 - p_2r_1) - xyf(p_1r_2 - p_2r_1)} \\ & 1(p_1r_2 - p_2r_1) - f(p_1r_2 - p_2r_1) - xy(p_1r_2 + p_2r_1) + xyf(p_1r_2 - p_2r_1) \end{aligned}$$

or

$$\frac{f(1 - xy)}{(1 - f)(1 - xy)}$$

or, as long as $1 - xy \neq 0$ (if $xy = 1$, we would have a single point with no slope),

$$f/(1-f)$$

which is identical to the previous slope. We expect this because this process is only a special case of a mixing line.

However, if we now allow our two magmas to re-differentiate with the same ratios of elements as in the differentiation of the original magma, we now have a slope of

$$\begin{aligned} & \frac{[d_1 + fp_1 + x(d_2 + fp_2)]/(1 + x)}{(r_1 + xr_2)/(1 + x)} - \frac{[d_2 + fp_2 + y(d_1 + fp_1)]/(1 + y)}{(r_2 + yr_1)/(1 + y)} \\ & \underline{\frac{p_1 - fp_1}{r_1} - \frac{p_2 - fp_2}{r_2}} \end{aligned}$$

or

$$\begin{aligned} & \frac{[d_1 + fp_1 + x(d_2 + fp_2)](r_2 + yr_1) - [d_2 + fp_2 + y(d_1 + fp_1)](r_1 + xr_2)}{(r_1 + xr_2)(r_2 + yr_1)} \\ & \underline{\frac{p_1r_2 - fp_1r_2 - p_2r_1 + fp_2r_1}{r_1r_2}} \end{aligned}$$

or

$$\begin{aligned} & d_1r_2 + fp_1r_2 + xd_2r_2 + xfp_2r_2 + yd_1r_1 + yfp_1r_1 + xyd_2r_1 + xyfp_2r_1 \\ & \underline{-d_2r_1 - fp_2r_1 - y(d_1r_1 - fp_1r_1 - xd_2r_2 - xfp_2r_2 - xyd_1r_2 - xyfp_1r_2)} \\ & \underline{\frac{(r_1 + xr_2)(r_2 + yr_1)}{p_1r_2 - p_2r_1 - fp_1r_2 + fp_2r_1}} \\ & r_1r_2 \end{aligned}$$

or

$$\frac{(d_1r_2 - d_2r_1) + (yd_1r_1 - yd_1r_1) + (yfp_1r_1 - yfp_1r_1) + (xyd_2r_1 - xyd_1r_2) + fp_1r_2 - fp_2r_1 + (xd_2r_2 - xd_2r_2) + (xfp_2r_2 - xfp_2r_2) + xyfp_2r_1 - xyfp_1r_2}{\frac{(r_1 + xr_2)(r_2 + yr_1)}{p_1r_2 - p_2r_1 - fp_1r_2 + fp_2r_1}} \cdot r_1r_2$$

or

$$\frac{f(p_1r_2 - fp_2r_1) + xyf(p_2r_1 - p_1r_2)}{(p_1r_2 - p_2r_1) - f(p_1r_2 - p_2r_1)} \cdot \frac{r_1r_2}{(r_1 + xr_2)(r_2 + yr_1)}$$

or

$$\frac{f}{1-f} (1-xy) \frac{r_1r_2}{r_1r_2 + xr_2r_2 + yr_1r_1 + xyr_1r_2}$$

or

$$\frac{f}{1-f} \frac{(1-xy)}{1 + xr_2/r_1 + yr_1/r_2 + xy}$$

This means that the slope is reduced by a factor that is dependent on x, y, and r_1/r_2 . If $xy = 1$, the age disappears. If $xy > 1$, the slope is a “futurechron”, giving an age in the future. Two different dating systems will give identical slope reductions if either r_1/r_2 is equal for both systems, or if $y = x(r_1/r_2)_1(r_1/r_2)_2$. This is because

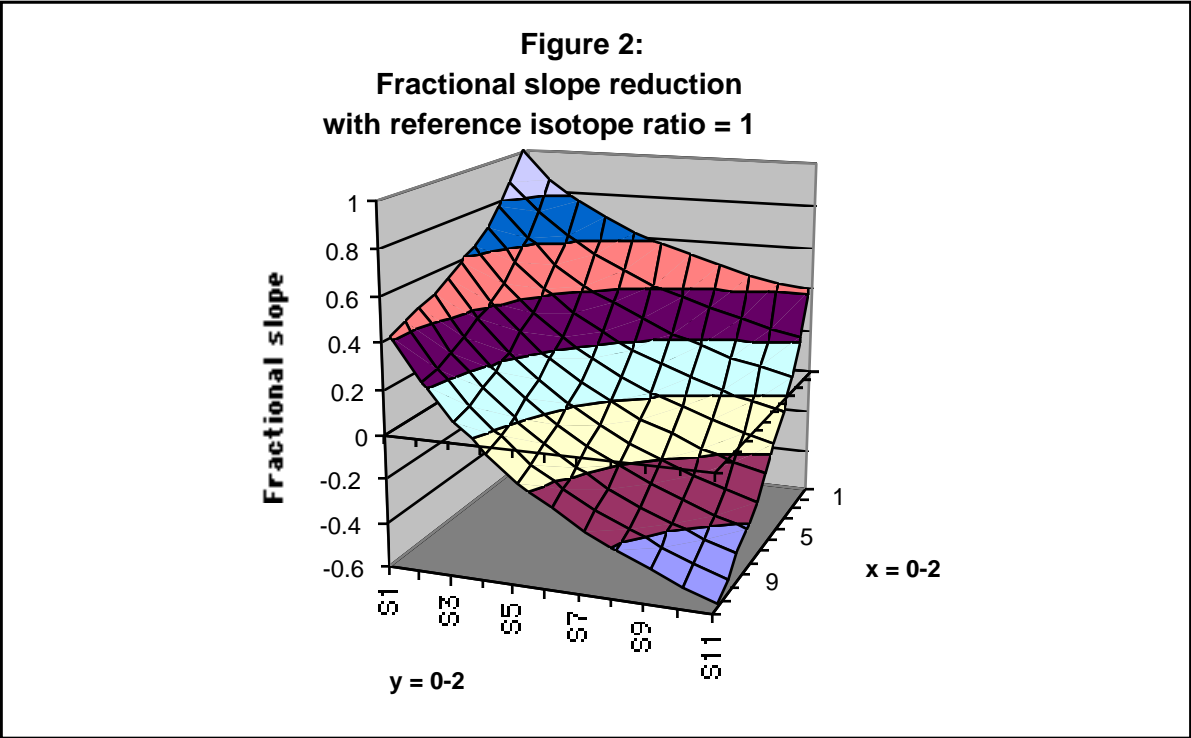
$$\begin{aligned} (1-xy)[1+x(r_1/r_2)_1+y/(r_1/r_2)_1+xy] &= (1-xy)[1+x(r_1/r_2)_2+y/(r_1/r_2)_2+xy] \\ 1+x(r_1/r_2)_1+y/(r_1/r_2)_1+xy &= 1+x(r_1/r_2)_2+y/(r_1/r_2)_2+xy \\ x(r_1/r_2)_1+y/(r_1/r_2)_1 &= x(r_1/r_2)_2+y/(r_1/r_2)_2 \\ x(r_1/r_2)_1-x(r_1/r_2)_2 &= y/(r_1/r_2)_2-y/(r_1/r_2)_1 \\ x(r_1/r_2)_1^2(r_1/r_2)_2-x(r_1/r_2)_1(r_1/r_2)_2^2 &= y(r_1/r_2)_1-y(r_1/r_2)_2 \\ x(r_1/r_2)_1(r_1/r_2)_2[(r_1/r_2)_1-(r_1/r_2)_2] &= y[(r_1/r_2)_1-(r_1/r_2)_2]. \end{aligned}$$

Assuming $(r_1/r_2)_1 \neq (r_1/r_2)_2$,

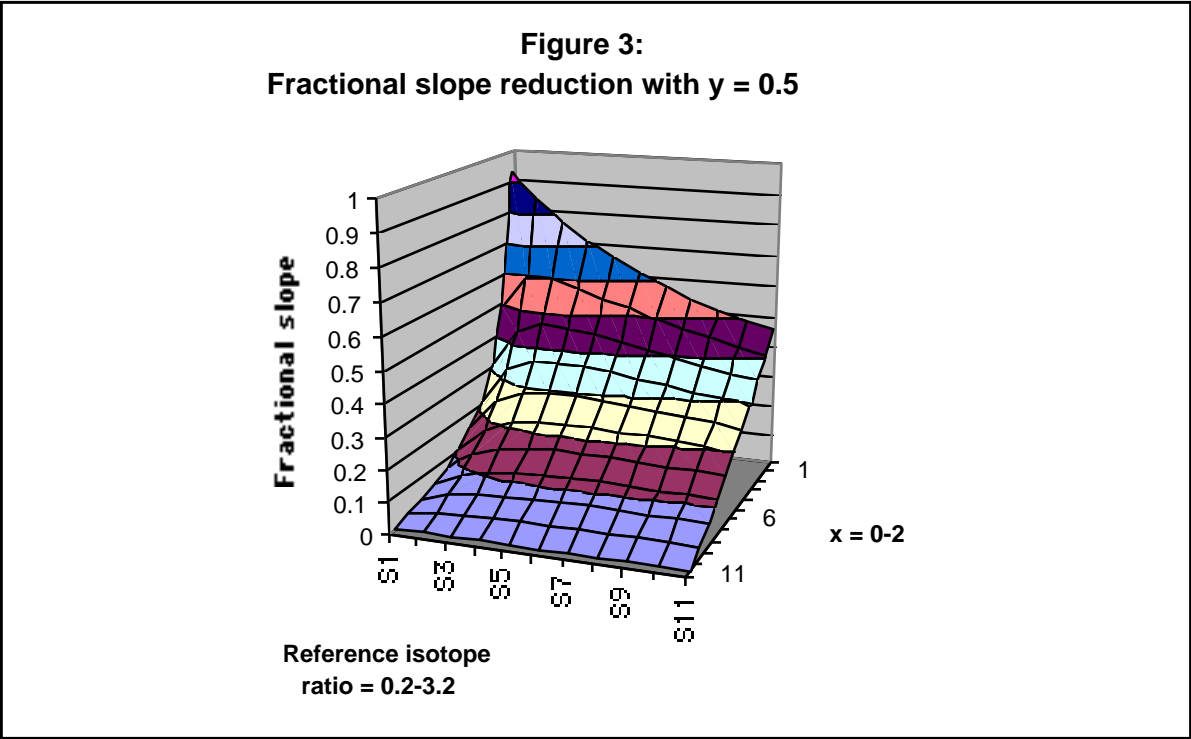
$$x(r_1/r_2)_1(r_1/r_2)_2 = y.$$

(If $x = y$, this means that $(r_1/r_2)_1 = (r_1/r_2)_2$.)

However, three further points need to be made. First, this model may prove in the end to be no more reliable than the one which posited a mixing hyperbola, about which Faure had his doubts. Second, it is not necessary for either the fractionation ratios to be identical or the x and y values to precisely match their differences. All that has to happen is that they be close enough so that the age reductions match to within the limits of statistical error. Some idea of the behavior of these variables may be obtained from the following observations. If x and y are varied while holding r constant one gets a graph something like the one on the next page.



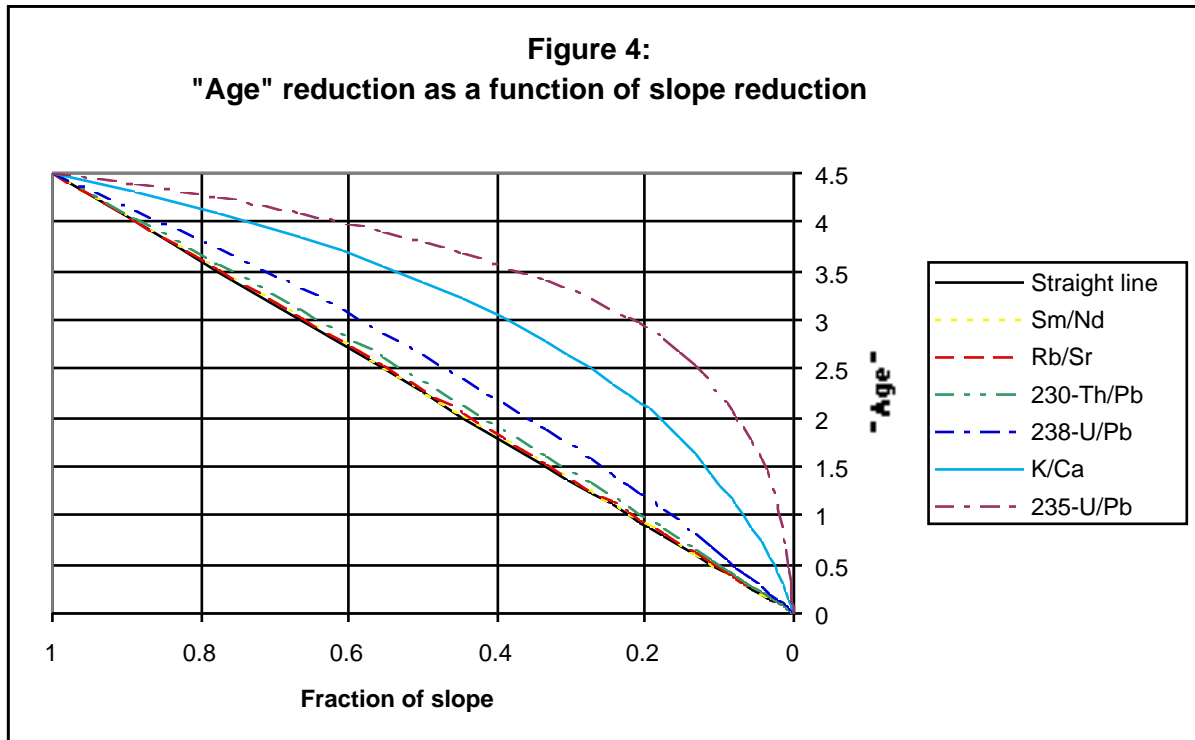
If one holds y constant while allowing x and r to vary, the results look like this:



Another complication is that slope reduction does not automatically translate linearly into “age” reduction. The formula for the reduction of an “age” is

$$t = \ln [1 + s(e^{\lambda a} - 1)] / \lambda,$$

where s is the reduced slope divided by the original slope and a is the “age” of the original “isochron”. The following is a graph of s versus t for $a = 4.5$ billion years for various dating systems.



Note that for samarium-neodymium and rubidium-strontium dating the transformation is nearly linear, whereas for uranium-238 dating the transformation is significantly non-linear, and for uranium-235 dating the transformation is grossly non-linear. It is of interest that usually uranium-235 dates are generally “older” than uranium-238 dates, which is predicted by this model. Also, neither generally match thorium-232 dates.

Finally, I will suggest that this model should be tested, first on whole-rock systems that everyone can agree are mixing lines, and then with systems where there is a disagreement between creationists and evolutionists about whether they are isochron lines or mixing lines. The results could conceivably be helpful in resolving the impasse between the two sides. Then again, the results may not, as the side that has more difficulty explaining the evidence may simply retrench. At least the exercise may help to elucidate more clearly the nature of the debate.

Having said all that, I do not believe that creationists should quit either believing in a short time for life on earth, or in the possibility that the presently available radiometric data can be explained using a short-age model, at least not at this time. Now is not the time for us to throw in the towel.